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A. G. Chentsov

Methods of Extremal Routing and Their Application to the Control of Sheet Cutting on CNC Machines

A. G. Chentsov

N. N. Krasovskii Institute of Mathematics and Mechanics,
Ural Branch of the Russian Academy of Sciences (16 S. Kovalevskoy St. Ekaterinburg. 620108. Russia).

B. N. Yeltsin Ural Federal University (19 Mira St. Ekaterinburg. 600002. Russia).

chentsov@imm.uran.ru

Abstract. An extremal routing problem with constraints and complicated cost functions is considered. The investigated setting is oriented to application in engineering problems connected with sheet cutting on the machines with CNC. Nonstandard variant of dynamic programming is used for construction of an optimal solution including the starting point, the route (index permutation), and a concrete trajectory of the process. This procedure is implemented in the form of standard programs for a PC and a supercomputer.

INTRODUCTION

Routing problems connected with engineering applications are considered. These problems are investigated in mathematical setting as extremal problems with constraints and complicated cost functions. Among all constraints, we select precedence conditions. Our cost functions can include the task list dependence. These singularities are typical under consideration of sheet cutting on machines with numerical program control (CNC). We keep in mind tool control during sheet cutting. For solving, we use the widely understood dynamic programming (DP). We find a global extremum and an optimal solution including the choice of the starting point, the route (index permutation), and a specific path concrete trajectory of the process.

The natural prototype of our routing problem is the known traveling salesman problem (TSP); see [1, 2, 3]. However, the considered routing problem contains many quality features (diffculties of the computing nature intrinsic to the TSP remain and, what is more, become worse). The diffculty of the CNC problem with respect to the TSP is the natural cause for wide application of heuristic algorithms; see [4, 5, 6, 7, 8, 9]. Therefore, the above-mentioned CNC problem demanded a serious theoretical investigation; see [10, 11, 12]. This approach goes back to the monograph [13].

MATHEMATICAL SETTING OF THE ROUTING PROBLEM

We use the general setting of the routing problem with constraints and (complicated) cost functions with the task list dependence. Fix nonempty sets X and X^0, where $X^0 \subset X$, a number $N \in \mathbb{N} \triangleq \{1;2;\ldots\}$ (here and further, $\triangleq$ is the equality by definition) for which $N \geqslant 2$. Moreover, fix the nonempty finite sets $M_1,\ldots,M_N$ for which $M_j \subset X$ under $j \in \overline{1,N} \triangleq \{k \in \mathbb{N} \mid k \leqslant N\}$. Finally, we fix nonempty sets $\mathbb{M}_1,\ldots,\mathbb{M}_N$ such that $\mathbb{M}_j \subset M_j \times M_j$ under $j \in \overline{1,N}$. Let $\mathbb{P}$ be the set of all permutations of the set $\overline{1,N}$. We consider processes

$$x^0 \longrightarrow (x_1^{(1)} \in M_{\alpha(1)} \rightsquigarrow x_2^{(1)} \in M_{\alpha(1)}) \to \ldots \to (x_1^{(N)} \in M_{\alpha(N)} \rightsquigarrow x_2^{(N)} \in M_{\alpha(N)}), \tag{1}$$

where $\alpha \in \mathbb{P}$, $x^0 \in X^0$, $(x_1^{(1)},x_2^{(1)}) \in \mathbb{M}_{\alpha(1)},\ldots,(x_1^{(N)},x_2^{(N)}) \in \mathbb{M}_{\alpha(N)}$. We must choose

$$(\alpha,x^0,(x_1^{(1)},x_2^{(1)}),\ldots,(x_1^{(N)},x_2^{(N)})),$$

where the choice of α must satisfy to precedence conditions. These conditions are defined by the set $\mathbf{K}$, $\mathbf{K} \subset \overline{1,N} \times \overline{1,N}$, for which [13] (Condition 2.2.1) holds. Then

$$\mathbf{A} \triangleq \{\alpha \in \mathbb{P} \mid \forall z \in \mathbf{K}\ \forall t_1 \in \overline{1,N}\ \forall t_2 \in \overline{1,N}\ \ (z = (\alpha(t_1),\alpha(t_2))) \Longrightarrow (t_1 < t_2)\}$$

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is the set of all admissible (by precedence) routes. Therefore, in (1), only variant $\alpha \in \mathbf{A}$ is admissible. However, only a route $\alpha \in \mathbf{A}$ does not defines the process development (see (1)). We must introduce trajectories coordinated with the route.

If z is an ordered pair (OP), then by $pr_1(z)$ and $pr_2(z)$ we denote the first and the second elements of z respectively. Under $j \in \overline{1,N}$, we introduce nonempty finite sets $\mathfrak{M}_j \stackrel{\triangle}{=} \{pr_1(z) : z \in \mathbb{M}_j\}$ and $\mathbf{M}_j \stackrel{\triangle}{=} \{pr_2(z) : z \in \mathbb{M}_j\}$; we consider $x \in \mathfrak{M}_j$ as a possible point of arrival and $y \in \mathbf{M}_j$ as a possible point of departure for M_j. In addition (see (1)), $(x,y) \in \mathbb{M}_j$. By $\mathfrak{M}$ and $\mathbf{M}$ we denote the unions of all sets $\mathfrak{M}_i$, $i \in \overline{1,N}$, and $\mathbf{M}_i$, $i \in \overline{1,N}$, respectively. Suppose that $\mathbb{X} \stackrel{\triangle}{=} X^0 \cup \mathfrak{M}$ and $\mathbf{X} \stackrel{\triangle}{=} X^0 \cup \mathbf{M}$, consider $\mathbb{X} \times \mathbf{X}$ as the phase space of our process (see (1)). Under $\overline{0,N} \stackrel{\triangle}{=} \{0\} \cup \overline{1,N}$ (here $\{0\}$ is the singleton containing 0), $\mathbb{Z}$ denotes the set of all mappings from $\overline{0,N}$ into $\mathbb{X} \times \mathbf{X}$. For $x \in X^0$ and $\alpha \in \mathbb{P}$ the set

$$\mathbb{Z}_\alpha[x] \stackrel{\triangle}{=} \{(z_t)_{t \in \overline{0,N}} \in \mathbb{Z} \mid (z_0 = (x,x)) \& (z_t \in \mathbb{M}_{\alpha(t)} \;\; \forall t \in \overline{1,N})\}$$

is the bundle of trajectories coordinated with the route α and starting from (x,x). Under $x \in X^0$, we consider

$$\tilde{\mathbf{D}}[x] \stackrel{\triangle}{=} \{(\alpha, \mathbf{z}) \in \mathbf{A} \times \mathbb{Z} \mid \mathbf{z} \in \mathbb{Z}_\alpha[x]\}$$

as the set of all admissible solutions for the starting point x. As a corollary,

$$\mathbf{D} \stackrel{\triangle}{=} \{(\alpha, \mathbf{z}, x) \in \mathbf{A} \times \mathbb{Z} \times X^0 \mid (\alpha, \mathbf{z}) \in \tilde{\mathbf{D}}[x]\}$$

is the set of all admissible solutions of the complete problem (this problem is formulated below).

Let $\mathbb{R}_+ \stackrel{\triangle}{=} \{\xi \in \mathbb{R} \mid 0 \leqslant \xi\}$ and $\mathscr{R}_+[T]$ be the set of all functions from a set T into $\mathbb{R}_+$. We fix $N+2$ functions

$$\mathbf{c} \in \mathscr{R}_+[\mathbf{X} \times \mathbb{X} \times \mathfrak{N}], c_1 \in \mathscr{R}_+[\mathbb{X} \times \mathbf{X} \times \mathfrak{N}], \ldots, c_N \in \mathscr{R}_+[\mathbb{X} \times \mathbf{X} \times \mathfrak{N}], f \in \mathscr{R}_+[\mathbf{X}],$$

where $\mathfrak{N}$ is the family of all nonempty subsets of $\overline{1,N}$. The function $\mathbf{c}$ is used for estimation of the exterior movements; $c_1, \ldots, c_N$ are used under estimation of interior works, and f estimates the terminal state (the point $x_2^{(N)}$ in (1)). We consider the additive criterion. For this, under $\alpha \in \mathbb{P}$ and $\mathbf{z} \in \mathbb{Z}_\alpha[x]$, suppose that

$$\begin{aligned} \mathfrak{C}_\alpha[\mathbf{z}] \stackrel{\triangle}{=} \sum_{s=1}^{N} [&\mathbf{c}(pr_2(\mathbf{z}(s-1)), pr_1(\mathbf{z}(s)), \{\alpha(k) : k \in \overline{s,N}\}) \\ &+ c_{\alpha(s)}(\mathbf{z}(s), \{\alpha(k) : k \in \overline{s,N}\})] + f(pr_2(\mathbf{z}(N))), \end{aligned} \tag{2}$$

where $\overline{m,n} \stackrel{\triangle}{=} \{j \in \mathbb{N}_0 \mid (m \leqslant j) \& (j \leqslant n)\}$ for $m \in \mathbb{N}_0$ and $n \in \mathbb{N}_0$. Under $x \in X^0$, in (2), we use the variant $\alpha \in \mathbf{A}$ and $\mathbf{z} \in \mathbb{Z}_\alpha[x]$. A more general case is realized when $(\alpha, \mathbf{z}, x) \in \mathbf{D}$. However, for the case $x \in X^0$, we obtain the next x-problem

$$\mathfrak{C}_\alpha[\mathbf{z}] \longrightarrow \min, (\alpha, \mathbf{z}) \in \tilde{\mathbf{D}}[x], \tag{3}$$

for which extremum $V[x] \in \mathbb{R}_+$ is defined. Let X^0 be a finite set. Along with (3), we consider the complete problem

$$\mathfrak{C}_\alpha[\mathbf{z}] \longrightarrow \min, (\alpha, \mathbf{z}, x) \in \tilde{\mathbf{D}}. \tag{4}$$

We denote by $\mathbb{V}$ the corresponding extremum of the problem (4). We consider (4) as main routing problem.

DYNAMIC PROGRAMMING (ECONOMICAL VARIANT)

To solve problems (3) and (4), we use the nonstandard variant of DP; see [10, 11, 12]. This procedure is the essential development of the scheme [13] (§4.9). In the considered article, we use only the algorithmic variant (we note that the yet more general setting is reduced in [14]; in particular, we note that in [14] (Theorem 5.1), the detailed proof of the Bellman equation is reduced). For this, we recall the mapping $\mathbf{I}$ operating in $\mathfrak{N}$ by the rule [13] ((2.2.28)) (in this connection, [13] ((2.2.26) and Proposition 2.2.3) is useful). This mapping defines the rule of deletion of tasks from a list. The mapping $\mathbf{I}$ was used in [10, 11, 12, 13, 14]. Later, we introduce the family

$$\mathscr{C} \stackrel{\triangle}{=} \{K \in \mathfrak{N} \mid \forall (x,y) \in \mathbf{K} \ \ (x \in K) \Longrightarrow (y \in K)\}$$

of all essential lists. Moreover, under $s \in \overline{1,N}$, we introduce

$$\mathscr{C}_s \stackrel{\triangle}{=} \{K \in \mathscr{C} \mid s = |K|\},$$

where $|\mathbb{K}| \in \mathbb{N}$ is finite set power of every nonempty finite set $\mathbb{K}$. Thus, $(\mathscr{C}_1, \ldots, \mathscr{C}_N)$ is a partition of $\mathscr{C}$, where $\mathscr{C}_N = \{\overline{1,N}\}$ (the singleton containing $\overline{1,N}$) and

$$\mathscr{C}_{k-1} = \{K \setminus \{t\} : K \in \mathscr{C}_k, t \in \mathbf{I}(K)\}$$

for $k \in \overline{2,N}$ (the set $\mathscr{C}_1$ is defined very simply; see [10, 11, 12, 13, 14]). Thus, we obtain $(\mathscr{C}_1, \ldots, \mathscr{C}_N)$ by the recurrence procedure. Later, we construct the sets $D_0, D_1, \ldots, D_N$; the elements of every of these sets are OP (x,K), where $x \in \mathbf{X}$ and K is a subset of $\overline{1,N}$. In addition, $D_N \stackrel{\triangle}{=} \{(x, \overline{1,N}) : x \in X^0\}$. Moreover, for $\mathbf{K}_1 \stackrel{\triangle}{=} \{pr_1(z) : z \in \mathbf{K}\}$, we have $D_0 \stackrel{\triangle}{=} \{(x, \varnothing) : x \in \tilde{\mathscr{M}}\}$, where $\tilde{\mathscr{M}}$ is the union of all sets $\mathbf{M}_t$, $t \in \overline{1,N} \setminus \mathbf{K}_1$.

Consider the construction of D_s under $s \in \overline{1,N-1}$. At first, under $K \in \mathscr{C}_s$, we introduce the sets $J_s(K)$, $\mathscr{M}_s[K]$, and $\mathbb{D}_s[K]$ by the rules of [11] (p.1963). Later, we suppose that D_s is the union of all sets $\mathbb{D}_s[K]$, $K \in \mathscr{C}_s$. Since the choice of s was arbitrary, we obtain all sets $D_1, \ldots, D_{N-1}$. As a corollary, the collection $(D_0, D_1, \ldots, D_N)$ is constructed.

Now, we construct functions

$$v_0 \in \mathscr{R}_+[D_0], v_1 \in \mathscr{R}_+[D_1], \ldots, v_N \in \mathscr{R}_+[D_N]$$

by recurrence procedure using the natural property

$$(pr_2(z), K \setminus \{j\}) \in D_{t-1} \ \ \forall t \in \overline{1,N} \ \ \forall (x,K) \in D_t \ \ \forall j \in \mathbf{I}(K) \ \ \forall z \in \mathbb{M}_j.$$

Namely, we suppose that $v_0(x, \varnothing) \stackrel{\triangle}{=} f(x) \ \ \forall x \in \tilde{\mathscr{M}}$. For $s \in \overline{1,N}$, the transformation of v_{s-1} into v_s defined by [11] (Proposition 4.1). Thus, we obtain the recurrence procedure

$$v_0 \longrightarrow v_1 \longrightarrow \ldots \longrightarrow v_{n-1} \longrightarrow v_N \tag{5}$$

(of course, in (5), it is supposed that $N \geqslant 4$; for $N = 3$, we have the procedure $v_0 \to v_1 \to v_2 \to v_3$). The main result consists in the following statement:

$$V[x] = v_N(x, \overline{1,N}) \ \ \forall x \in X^0.$$

As a corollary, by v_N we can find the global extremum $\mathbb{V}$: $\mathbb{V}$ is the least of numbers $v_N(x, \overline{1,N})$, $x \in X^0$. Moreover, we find $x^0 \in X^0$ such that $v_N(x^0, \overline{1,N}) = \mathbb{V}$ (we recall that X^0 is a finite set). Thus, x^0 is the optimal starting point. We note that, for the construction of only v_N and $\mathbb{V}$, we can use the variant procedure with overwriting functions-layers v_j, $j \in \overline{1,N}$, of the Bellman function. Under this overwriting procedure, in the computer memory, only one function from (5) is situated. Yet, to build an optimal solution, all functions used in (5) are required. Namely, if all functions (5) are constructed, we can construct the optimal solution $(\alpha^0, \mathbf{z}^0) \in \tilde{\mathbf{D}}[x^0]$ (so, $\mathfrak{C}_{\alpha^0}[\mathbf{z}^0] = V[x^0]$) by the standard (for DP) retrograde procedure (see, for example, [11] (Section 4)). As a result, the triplet $(\alpha^0, \mathbf{z}^0, x^0) \in \mathbf{D}$ is the optimal solution of problem (4): $\mathfrak{C}_{\alpha^0}[\mathbf{z}^0] = \mathbb{V}$.

COMPUTING REALIZATION

The above-mentioned optimal algorithm was realized as standard program for PC and for supercomputer. For this, a parallel algorithm was constructed. The corresponding theoretical basis of this algorithm is the scheme of independent calculations from [15, 16, 17]. We note some typical dimensional indexes achieved in computing experiment. Namely, with the use of PC, routing problems of the above-mentioned type with 35 megalopolises were solved (program of P.A. Chentsov). In addition, the case $|\mathbf{K}| = 24$ was considered. This algorithm was used under dynamic constraints for a problem connected with sheet cutting. Calculations with of a supercomputer were carried out for solving another applied problem. Namely, we consider the problem of dismantling radiating elements. Megalopolises are realized by digitization the boundaries of the near zone of radiating sources. The variant with 50 30-element megalopolises was investigated; in this case an optimal solution was obtained (program of A.M.Grigor'ev). Thus the DP-procedure can be used to solve actual engineering problems with elements of routing.

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